THE QUADRATIC FORMULA

LEARNING GOALS

- Learn how to use the quadratic formula to find the roots of a quadratic equation.

DERIVING THE FORMULA

Example	Quadratic Formula
$2x^2 + 5x + 1 = 0$	$ax^2 + bx + c = 0$
$x^2 + \frac{5}{2}x + \frac{1}{2} = 0$	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
$\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) + \frac{1}{2}$	$\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + \frac{c}{a}$
$\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right) - \left(\frac{5}{4}\right)^2 + \frac{1}{2}$	$\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$
$\left(\quad\right)^2 - \frac{25}{16} + \frac{1}{2}$	$\left(\quad\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}$
$\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{8}{16}$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}$
$\left(x + \frac{5}{4}\right)^2 - \frac{17}{16} = 0$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 + 4ac}{4a^2} = 0$
$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{\frac{17}{16}}$	$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$
$x + \frac{5}{4} = \pm\frac{\sqrt{17}}{4}$	$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{5}{4} \pm \frac{\sqrt{17}}{4}$	$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-5 \pm \sqrt{17}}{4}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{5 + \sqrt{17}}{4}$ $x = -\frac{5 - \sqrt{17}}{4}$	$x = -\frac{b + \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$-2ab = 5x$, $b = \frac{5}{2}x$

Vertex $(x - h)^2$

THE QUADRATIC FORMULA

Definition

- a formula for determining the roots of a quadratic equation in standard form.

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1: REAL ROOTS

Use the quadratic formula to solve each quadratic equation. Where necessary, round to the nearest hundredth.

$2x^2 + 9x + 6 = 0$

$a = 2 \quad b = 9 \quad c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-9 \pm \sqrt{9^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{-9 + \sqrt{9^2 - 4(2)(6)}}{2(2)} = -0.8$$

$$x = \frac{-9 - \sqrt{9^2 - 4(2)(6)}}{2(2)} = -3.7$$

2 Solutions

$4x^2 - 12x = -9$

$4x^2 - 12x + 9 = 0$

$a = 4 \quad b = -12 \quad c = +9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$

$x = 1.5$

1 Solution.

EXAMPLE 2: USE THE QUADRATIC FORMULA TO SKETCH A PARABOLA

Find the x-intercepts, the vertex, and the equation of the axis of symmetry of the quadratic relation $y = -5x^2 + 8x - 3$. Sketch the Parabola.

$y = -5x^2 + 8x - 3$

$a = -5 \quad b = 8 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(-5)(-3)}}{2(-5)}$$

$x = 0.6, 1$

$r \quad s$

$V(h, k)$

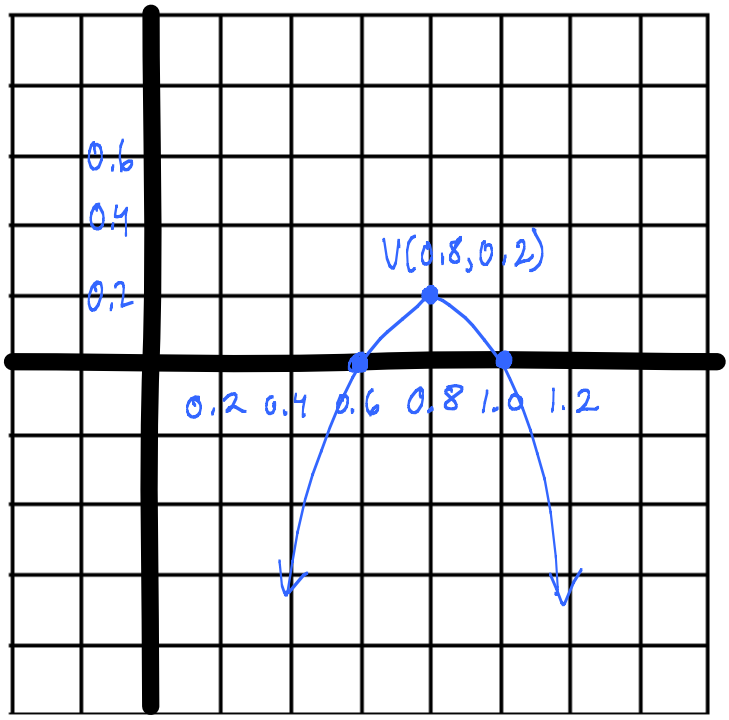
$$h = \frac{r + s}{2}$$

$= 0.8$

$k = -5h^2 + 8h - 3$

$= 0.2$

$V(0.8, 0.2)$



EXAMPLE 3: CONNECT A PARABOLA AND NO REAL ROOTS

A parabola has equation $y = (x-2)^2 + 3$.

a) State the coordinates of the vertex, the equation of the axis of symmetry, and the direction of opening.

V(2, 3) opens up.

b) Determine the x-intercepts. Verify using the quadratic formula.

$y = (x-2)(x-2) + 3$

$= x^2 - 2x - 2x + 4 + 3$

$= x^2 - 4x + 7$

$a = 1 \quad b = -4 \quad c = 7$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

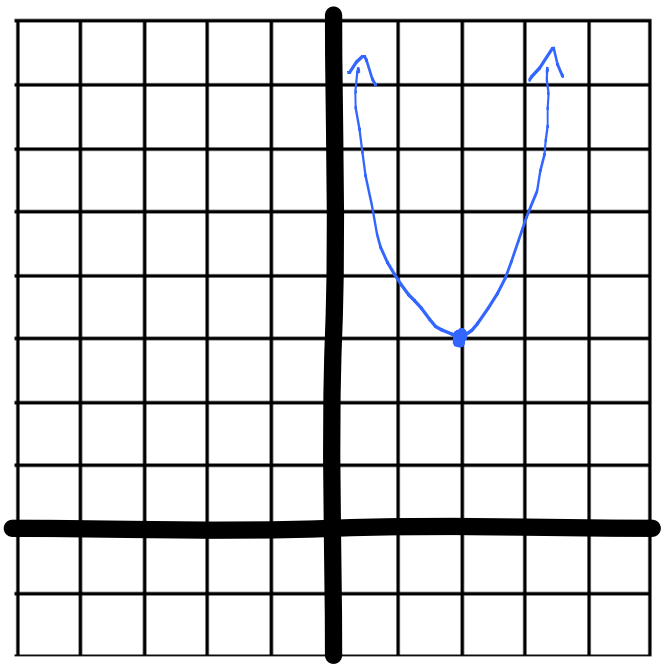
$= \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16 - 28}}{2}$

$\therefore \sqrt{-12}$ is not a real number

$\therefore$ there are no real roots.

c) Sketch the parabola



EXAMPLE 4: PATH OF A BASKETBALL

The path of a basketball after it is thrown from a height of 1.5m above the ground if given by the equation $h = -0.25d^2 + 2d + 1.5$ where h is the height, in metres, and d is the horizontal distance in metres.

a) How far has the ball travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?

$h = 0$

$0 = -0.25d^2 + 2d + 1.5$ $\quad x = -0.69, 8.7$

$a = -0.25 \quad b = 2 \quad c = 1.5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(-0.25)(1.5)}}{2(-0.25)}$$

∴ it lands at a horizontal distance of 8.7m.

b) Find the horizontal distance when the basketball is at a height of 4.5m above the ground.

$h = 4.5$

$4.5 = -0.25d^2 + 2d + 1.5$

$0 = -0.25d^2 + 2d + 1.5 - 4.5$

$0 = -0.25d^2 + 2d - 3$

$a = -0.25 \quad b = 2 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = 2, 6$

∴ the basketball reaches this height at both 2m and 6m.

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